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EFFECTS OF STEPWISE VARIATION OF ALBEDO ON REFLECTIVITY AND TRANSMISSIVITY OF AN ISOTROPICALLY SCATTERING SLAB

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Abstract—The reflectivity and transmissivity of an isotropically scattering plane-parallel slab with stepwise variation of the single scattering albedo within the medium are determined for an isotropically incident radiation. Tabulated results, covering a wide range of albedos, are presented.

NOMENCLATURE

A_α	defined by equation (6a)
$\alpha_{i,\alpha}$	unknown expansion coefficient of layer i
B_α	defined by equation (6b)
$b_{i,\alpha}$	unknown expansion coefficient of layer i
$I_i(\tau,\mu)$	radiation intensity in layer i
M	number of layers
N	the order of approximation
q^+, q^-	forward and backward radiation fluxes

Greek symbols

μ	the cosine of the angle between the direction of radiation intensity and the τ axis
ξ_i	discrete eigenvalues
τ	optical variable
τ_0	optical thickness
ϕ	eigenfunction
ω	single scattering albedo

Subscript

i	number of the layer

1. INTRODUCTION

THE REFLECTIVITY and transmissivity of a slab for an externally incident isotropic radiation have been studied extensively for the case of uniform albedo within the medium [1-6]. There are situations in which the single scattering albedo varies with the position within the medium. For example, a layer of turbid water in which turbidity varies with depth; porous materials such as fibers, powders used as lightweight insulators in which the porosity varies with depth; rocket exhaust gas containing a significant amount of micron-size particles, the concentration of which varies with the position in the flow; and many others. In all such cases, the radiation is scattered within the medium in addition being absorbed and emitted by the medium. It is of interest to predict the effect of variation of albedo within the medium on the reflectivity and transmissivity of a region of finite optical thickness. Therefore, the objective of this work is to present reflectivity and transmissivity data over a wide range of practical cases, in order to estimate the effects of the variation of albedo within the medium on reflectivity and transmissivity. To achieve this objective, four different classes of problems, schematically illustrated in Fig. 1, are considered.

Case 1

This represents the standard problem of a slab of optical thickness τ_0, having a uniform albedo ω throughout the region, subjected to externally incident isotropic radiation on the boundary surface at $\tau = 0$.

Case 2

This represents a situation in which a slab of optical thickness τ_0 has an albedo ω_1 for one half of the optical thickness (i.e. $0 \leqslant \tau \leqslant \frac{1}{2}\tau_0$) and an albedo ω_2 for the other half.

Case 3

Here the slab of optical thickness τ_0 has three different albedos ω_1, ω_2 and ω_3 for the first one-third, second one-third and the last one-third portion of the slab, respectively.

Case 4

The slab of optical thickness τ_0 has four different albedos $\omega_1, \omega_2, \omega_3$ and ω_4 for the first, second, third and fourth quarters of the slab, respectively.

2. ANALYSIS

The problem of radiative transfer in a plane-parallel slab with stepwise variation of albedo within the medium can be regarded as a radiation problem for an M-layer slab with transparent boundaries. Therefore, we consider an M-layer slab of optical thickness τ_0, irradiated at the boundary surface $\tau = 0$ with an isotropic radiation of unit intensity. The mathematical formulation of this radiation problem is obtained by generalizing the two-layer problem, considered in ref.

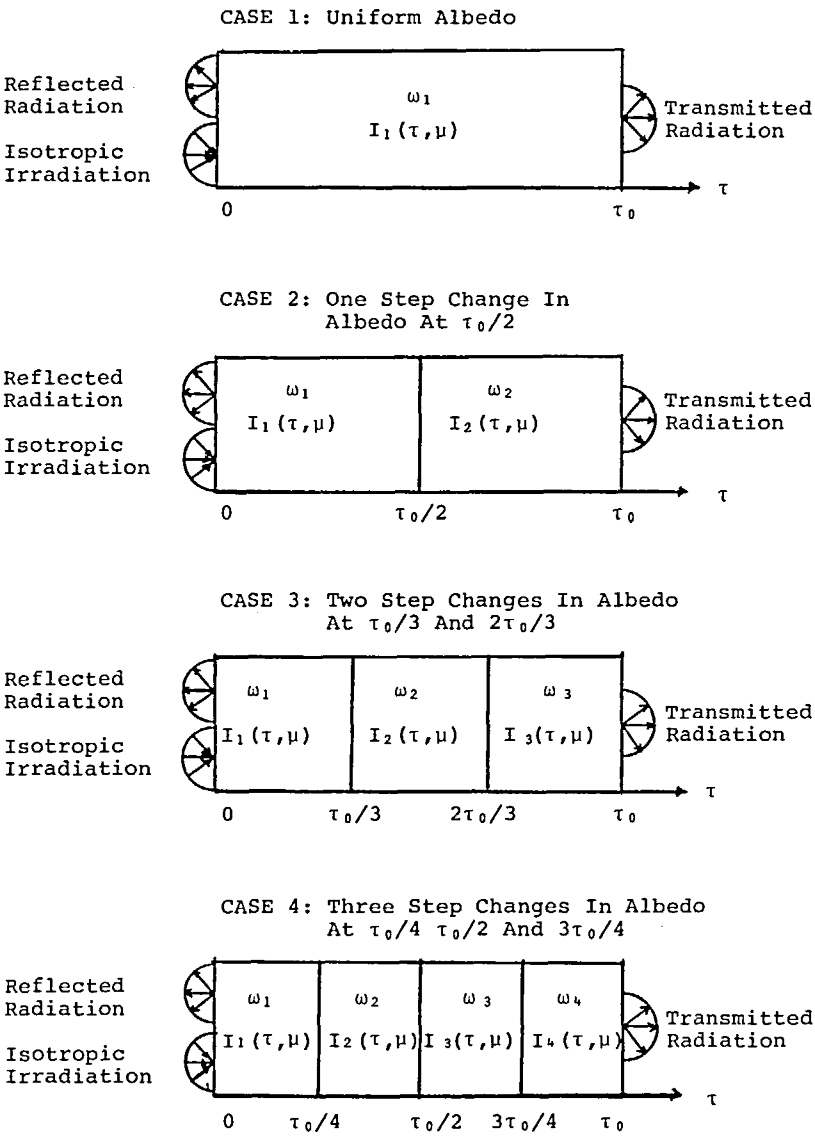

FIG. 1. The four cases considered.

[7], to an M-layer problem,

$$\mu \frac{\partial I_i(\tau,\mu)}{\partial \tau} + I_i(\tau,\mu) = \frac{\omega_i}{2} \int_{-1}^{1} I_i(\tau,\mu)\,\mathrm{d}\mu, \tag{1a}$$

in

$$(i-1)\frac{\tau_0}{M} \leqslant \tau \leqslant i\frac{\tau_0}{M}, \quad -1 \leqslant \mu \leqslant 1, \quad i = 1,2,\ldots,M,$$

$$I_1(0,\mu) = 1, \tag{1b}$$

$$I_M(\tau_0,-\mu) = 0, \tag{1c}$$

$$I_i\left(i\frac{\tau_0}{M},\mu\right) = I_{i+1}\left(i\frac{\tau_0}{M},\mu\right); \quad i = 1,2,\ldots,M-1, \tag{1d}$$

$$I_i\left(i\frac{\tau_0}{M},-\mu\right) = I_{i+1}\left(i\frac{\tau_0}{M},-\mu\right); \quad i = 1,2,\ldots,M-1. \tag{1e}$$

Here, μ is the cosine of the angle between the positive τ axis and the direction of radiation intensity, ω_i is the single scattering albedo for the region i and τ is the optical variable.

The general solution of this problem for any layer i can be written in exactly the same form as that given in ref. [1] (p. 355) for a single layer,

$$I_i(\tau,\mu) = C_i(\xi_i)\phi_i(\xi_i,\mu)\,\mathrm{e}^{-\tau/\xi_i} + C_i(-\xi_i)\phi_i(-\xi_i,\mu)\,\mathrm{e}^{\tau/\xi_i} + \int_{-1}^{1} C_i(\xi)\phi_i(\xi,\mu)\,\mathrm{e}^{-\tau/\xi}\,\mathrm{d}\xi \tag{2}$$

where $\phi_i(\pm\nu,\mu)$, $\nu \equiv \xi_i$ or $\xi(-1,1)$ is the eigenfunction; ξ_i is the discrete eigenvalue; and $C_i(\pm\nu)$ is the unknown expansion coefficients. We note that the solutions (2) satisfy the equation of radiative transfer exactly, but they involve the unknown expansion coefficients.

The full-range orthogonality property of the eigenfunctions $\phi_i(\nu, \mu)$ can be used to transform equations (2) into a system of singular integral equations, as will now be described.

For each layer i, equation (2) is evaluated at the left and right boundary, and the resulting equations are operated on first by the operator (ref. [1], p. 358)

$$\int_{-1}^{1} \mu\phi_i(-\nu, \mu)\,\mathrm{d}\mu, \quad \text{where } 0 \leqslant \nu \leqslant 1 \quad \text{or } \nu = \xi_i, \tag{3a}$$

and then by the operator

$$\int_{-1}^{1} \mu\phi_i(\nu, \mu)\,\mathrm{d}\mu, \quad \text{where } 0 \leqslant \nu \leqslant 1 \quad \text{or } \nu = \xi_i \tag{3b}$$

and the full-range orthogonality relations of the eigenfunctions are utilized, and the expansion coefficients are eliminated between the expressions. Finally, the integrals over the full range of μ appearing in these equations are split up into two parts involving only the half-range of μ, and the relation $\phi_i(-\nu, \mu) = \phi_i(\nu, -\mu)$ is utilized. The following system of $2M$ singular integral equations result:

$$\begin{aligned}\int_0^1 \mu\phi_i(-\nu, \mu) I_i\left[(i-1)\frac{\tau_0}{M}, \mu\right]\mathrm{d}\mu &- \int_0^1 \mu\phi_i(\nu, \mu) I_i\left[(i-1)\frac{\tau_0}{M}, -\mu\right]\mathrm{d}\mu \\ &- \mathrm{e}^{-\tau_0/M\nu}\int_0^1 \mu\phi_i(-\nu, \mu) I_i\left(i\frac{\tau_0}{M}, \mu\right)\mathrm{d}\mu \\ &+ \mathrm{e}^{-\tau_0/M\nu}\int_0^1 \mu\phi_i(\nu, \mu) I_i\left(i\frac{\tau_0}{M}, \mu\right)\mathrm{d}\mu = 0\end{aligned} \tag{4a}$$

and

$$\begin{aligned}\int_0^1 \mu\phi_i(\nu, \mu) I_i\left[(i-1)\frac{\tau_0}{M}, \mu\right]\mathrm{d}\mu &- \int_0^1 \mu\phi_i(-\nu, \mu) I_i\left[(i-1)\frac{\tau_0}{M}, \mu\right]\mathrm{d}\mu \\ &- \mathrm{e}^{-\tau_0/M\nu}\int_0^1 \mu\phi_i(\nu, \mu) I_i\left(i\frac{\tau_0}{M}, \mu\right)\mathrm{d}\mu \\ &+ \mathrm{e}^{\tau_0/M\nu}\int_0^1 \mu\phi_i(-\nu, \mu) I_i\left(i\frac{\tau_0}{M}, -\mu\right)\mathrm{d}\mu = 0\end{aligned} \tag{4b}$$

for $i = 1, 2, \ldots, M$.

We note that in these equations, the exit distributions $I_i[(i-1)(\tau_0/M), -\mu]$ and $I_i[i(\tau_0/M), \mu]$ for each layer i are the unknowns. Up to this point our analysis has been exact. However, we do not try to solve such a system of integral equations, but instead apply the basic concepts of the F_N method and represent the exit distributions by polynomials in μ in the form

$$I_i\left[(i-1)\frac{\tau_0}{M}, -\mu\right] = \sum_{\alpha=0}^{N} a_{i,\alpha}\mu^\alpha, \quad \mu \geqslant 0, \tag{5a}$$

$$I_i\left(i\frac{\tau_0}{M}, \mu\right) = \sum_{\alpha=0}^{N} b_{i,\alpha}\mu^\alpha, \quad \mu \geqslant 0. \tag{5b}$$

In addition, the following two definitions are introduced:

$$A_\alpha(\gamma) = \frac{2}{\omega_i\gamma}\int_0^1 \mu^{\alpha+1}\phi_i(-\gamma, \mu)\,\mathrm{d}\mu, \tag{6a}$$

$$B_\alpha^{(i)}(\gamma) = \frac{2}{\omega_i\gamma}\int_0^1 \mu^{\alpha+1}\phi_i(\gamma, \mu)\,\mathrm{d}\mu. \tag{6b}$$

When the polynomial representations (5a, b) are introduced into the integral equations (4), the boundary conditions for the problem (1) are applied and the definitions (6) are utilized, the following system of equations is obtained:

From equation (4a) for $i = 1$:

$$A_0(\nu) - \sum_{\alpha=0}^{N} [a_{1,\alpha}B_\alpha^1(\nu) + \mathrm{e}^{-\tau_0/M\nu}b_{1,\alpha}A_\alpha(\nu) - \mathrm{e}^{-\tau_0/M\nu}a_{2,\alpha}B_\alpha^1(\nu)] = 0. \tag{7a}$$

From equation (4b) for $i = 1$:

$$\mathrm{e}^{-\tau_0/M\nu}B_0^1(\nu) - \sum_{\alpha=0}^{N} [a_{1,\alpha}\,\mathrm{e}^{-\tau_0/M\nu}A_\alpha(\nu) + b_{1,\alpha}B_\alpha^1(\nu) - a_{2,\alpha}A_\alpha(\nu)] = 0. \tag{7b}$$

From equation (4a) for $i = 2, 3, \ldots, M-1$:

$$\sum_{\alpha=0}^{N} [b_{i-1,\alpha}A_\alpha(\nu) - a_{i,\alpha}B_\alpha^i(\nu) - \mathrm{e}^{-\tau_0/M\nu}b_{i,\alpha}A_\alpha(\nu) + \mathrm{e}^{-\tau_0/M\nu}a_{i+1,\alpha}B_\alpha^i(\nu)] = 0. \tag{7c}$$

From equation (4b) for $i = 2, 3, \ldots, M-1$:

$$\sum_{\alpha=0}^{N} [b_{i-1,\alpha}B_\alpha^i(\nu)\,\mathrm{e}^{-\tau_0/M\nu} - a_{i,\alpha}\,\mathrm{e}^{-\tau_0/M\nu}A_\alpha(\nu) - b_{i,\alpha}B_\alpha^i(\nu) + a_{i+1,\alpha}A_\alpha(\nu)] = 0. \tag{7d}$$

From equation (4a) for $i = M$:

$$\sum_{\alpha=0}^{N} [b_{M-1,\alpha}A_\alpha(\nu) - a_{M,\alpha}B_\alpha^M(\nu) - \mathrm{e}^{-\tau_0/M\nu}b_{M,\alpha}A_\alpha(\nu)] = 0 \tag{7e}$$

and finally from equation (4b) for $i = M$:

$$\sum_{\alpha=0}^{N} [\mathrm{e}^{-\tau_0/M\nu}b_{M-1,\alpha}B_\alpha^M(\nu) - a_{M,\alpha}\,\mathrm{e}^{-\tau_0/M\nu}A_\alpha(\nu) - b_{M,\alpha}B_\alpha^M(\nu)] = 0. \tag{7f}$$

It can be shown that the quantities $A_\alpha(\gamma)$ and $B_\alpha^i(\gamma)$ can be determined from the following recursive relations:

$$A_\alpha(\gamma) = -\gamma A_{\alpha-1}(\gamma) + \frac{1}{\alpha+1} \tag{8a}$$

with

$$A_0(\gamma) = 1 - \gamma\,\log_e\left(1 + \frac{1}{\gamma}\right) \tag{8b}$$

and

$$B_\alpha^i(\gamma) = \gamma B_{\alpha-1}^i(\gamma) - \frac{1}{\alpha+1} \tag{9a}$$

with

$$B_0^i(\gamma) = A_0(\gamma) + \frac{2}{\omega_i} - 2 \tag{9b}$$

and

$$i = 1, 2, \ldots, M.$$

The above set of equations (7a)–(7f) provide $2M(N+1)$ algebraic equations for the equal number of unknown expansion coefficients $a_{i,\alpha}$ and $b_{i,\alpha}$. Once these coefficients are computed, several quantities of practical interest are determined from their definitions. For example, the exit distribution of radiation intensity at $\tau = 0$ and $\tau = \tau_0$ are determined from

$$I_1(0, -\mu) = \sum_{\alpha=0}^{N} a_{1,\alpha}\mu^\alpha, \tag{10a}$$

$$I_M(\tau_0, \mu) = \sum_{\alpha=0}^{N} b_{M,\alpha}\mu^\alpha \tag{10b}$$

and the exit radiation heat fluxes from

$$q^-(0) = 2\pi \int_0^1 \mu I_1(0, -\mu)\,d\mu = 2\pi \sum_{\alpha=0}^{N} \frac{a_{1,\alpha}}{\alpha+2}, \tag{11a}$$

$$q^+(\tau_0) = 2\pi \int_0^1 \mu I_M(\tau_0, \mu)\,d\mu = 2\pi \sum_{\alpha=0}^{N} \frac{b_{M,\alpha}}{\alpha+2}. \tag{11b}$$

The reflectivity and the transmissivity of the slab are given by

$$\text{reflectivity} = \frac{q^-(0)}{q^+(0)} = 2 \sum_{\alpha=0}^{N} \frac{a_{1,\alpha}}{\alpha+2},$$

$$\text{transmissivity} = \frac{q^-(\tau_0)}{q^+(0)} = 2 \sum_{\alpha=0}^{N} \frac{b_{M,\alpha}}{\alpha+2}$$

since

$$q^+(0) = 2\pi \int_0^1 \mu I_1(0, \mu)\,d\mu = \pi$$

and both boundaries of the slab are transparent.

To perform the calculations, equations (7) are calculated at $N+1$ different values of v. In selecting the values of v, one point is taken as the eigenvalue associated with that step and the remaining values of v are determined from the relation $v = 2j - 1/2N$, for $j = 1, 2, \ldots, N$, where N is the order of the approximation.

3. RESULTS

The hemispherical reflectivity and transmissivity of slabs of various optical thicknesses from $\tau_0 = 0.1$ to $\tau_0 = 10.0$ are determined for 1–4 step changes in albedo for each optical thickness. The numerical results were obtained with an F_7 approximation and compared with the few available exact solutions [1, 5, 7, 8] of the single and two-layer problems. They were within ± 0.0001 of the exact results, except for $\tau_0 = 0.1$, which differed by no more than ± 0.0002.

Figure 1 illustrates the four different classes of problems considered in the present study. Table 1 summarizes the results of the reflectivity and transmissivity calculations for each of these four cases. The range of optical thicknesses covered varied from $\tau_0 = 0.1$ to $\tau_0 = 10$. All possible combinations of three different albedos, $\omega = 0.2, 0.8$ and 0.995, were used for each optical thickness considered.

Consider, for example, the case in which there are three step changes in albedo, given by $\omega_1 = 0.2$, $\omega_2 = 0.8$ and $\omega_3 = 0.995$ for the optical thickness $\tau_0 = 5$. The reflectivity and transmissivity of the slab for this particular case are given, respectively, by 0.04965 and 0.01739. The results for other combinations of ω_i can be determined in a similar manner. Therefore, the effects of stepwise variation of albedo within a slab on the reflectivity and transmissivity can be estimated by using the results given in Table 1.

The present results are also helpful in estimating the magnitude of the error involved in reflectivity and transmissivity calculations when a simple average of the albedo is considered instead of breaking the system into a series of discrete elements in order to take into account the variation of albedo within the medium. Consider for example a slab of optical thickness $\tau_0 = 2$ for the two different arrangements of step changes in albedo, given by 0.2–0.8 and 0.8–0.2, as illustrated in Case 2 of Fig. 1. For both of these cases the arithmetic average of the albedo is 0.5. The reflectivities for the cases 0.2–0.8 and 0.8–0.2 are respectively 0.05816 and 0.28698, whereas the reflectivity for the case $\omega = 0.5$ is 0.1071. Clearly, significant error is involved in the estimation of reflectivity if a simple arithmetic average is employed.

The transmissivities for the cases 0.2–0.8 and 0.8–0.2 are, respectively, 0.11460 and 0.11460; and for the case of arithmetic average $\omega = 0.5$ it is 0.1071. Clearly, in the case of transmissivity, the result with the arithmetic average of the albedos appears to be a good first approximation.

We compared the transmissivity and reflectivity obtained by taking a simple average of the albedos with the results given in Table 1 for several other combinations of albedos. It appears that, the use of simple arithmetic average for albedos can cause appreciable error in the calculation of reflectivity; but it may provide a good first approximation for the transmissivity.

The case of $\omega = 1$ is not included in the present analysis, because it requires a completely different formulation of the problem from the one considered here.

The total computer time for all the results presented in Table 1 was about 20 min on an IBM 3081 computer using G level FORTRAN.

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Table 1. Reflectivity and transmissivity of a slab

Albedo			$\tau_0 = 0.1$		$\tau_0 = 0.5$		$\tau_0 = 1.0$		$\tau_0 = 2.0$		$\tau_0 = 5.0$		$\tau_0 = 10.0$	
W1			REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY
0.200			0.01462	0.84684	0.03715	0.47435	0.04393	0.24626	0.04607	0.07274	0.04626	0.00245	0.04626	0.00001
0.800			0.06498	0.89651	0.20560	0.62199	0.28015	0.41624	0.32795	0.19727	0.34168	0.02292	0.34187	0.00065
0.995			0.08383	0.91517	0.29323	0.70179	0.44124	0.54884	0.59880	0.38154	0.76360	0.18923	0.82841	0.08667
W1	W2		REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY
0.200	0.200		0.01462	0.84684	0.03715	0.47435	0.04393	0.24626	0.04607	0.07274	0.04626	0.00245	0.04626	0.00001
0.200	0.800		0.03521	0.87064	0.07727	0.53834	0.07535	0.31337	0.05816	0.11460	0.04668	0.00684	0.04627	0.00007
0.200	0.995		0.04248	0.87904	0.09478	0.56644	0.09205	0.34990	0.06683	0.14737	0.04718	0.01538	0.04627	0.00055
0.800	0.200		0.04230	0.87061	0.14723	0.53835	0.21843	0.31337	0.28698	0.11460	0.33591	0.00684	0.34170	0.00007
0.800	0.800		0.06498	0.89651	0.20560	0.62199	0.28015	0.41624	0.32795	0.19727	0.34168	0.02292	0.34187	0.00065
0.800	0.995		0.07300	0.90569	0.23166	0.65956	0.31465	0.47513	0.36029	0.26865	0.34990	0.05990	0.34221	0.00610
0.995	0.200		0.05208	0.87904	0.19513	0.56644	0.31015	0.34990	0.45366	0.14737	0.65051	0.01538	0.76518	0.00055
0.995	0.800		0.07553	0.90570	0.26274	0.65956	0.39339	0.47513	0.53196	0.26866	0.69013	0.05990	0.77939	0.00610
0.995	0.995		0.08383	0.91517	0.29323	0.70179	0.44124	0.54884	0.59880	0.38154	0.76360	0.18923	0.82841	0.08667
W1	W2	W3	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY	REFLECTIVITY	TRANSMISSIVITY
0.200	0.200	0.200	0.01462	0.84684	0.03715	0.47435	0.04393	0.24626	0.04607	0.07274	0.04626	0.00245	0.04626	0.00001
0.200	0.200	0.800	0.02749	0.86250	0.05850	0.51529	0.05810	0.28805	0.05009	0.09777	0.04632	0.00482	0.04626	0.00004
0.200	0.200	0.995	0.03193	0.86790	0.06712	0.53187	0.06476	0.30788	0.05249	0.11324	0.04637	0.00765	0.04626	0.00013
0.200	0.800	0.200	0.03017	0.86236	0.07677	0.51340	0.08429	0.28507	0.07058	0.09483	0.04905	0.00432	0.04633	0.00003
0.200	0.800	0.800	0.04393	0.87900	0.10443	0.56345	0.10658	0.34235	0.07947	0.13541	0.04934	0.00983	0.04633	0.00014
0.200	0.800	0.995	0.04869	0.88477	0.11576	0.58402	0.11741	0.37043	0.08516	0.16232	0.04965	0.01739	0.04633	0.00055
0.200	0.995	0.200	0.03554	0.86773	0.09281	0.52923	0.10335	0.30359	0.08532	0.10862	0.05177	0.00655	0.04642	0.00009
0.200	0.995	0.800	0.04962	0.88472	0.12330	0.58326	0.13026	0.36913	0.09842	0.16060	0.05263	0.01670	0.04643	0.00050
0.200	0.995	0.995	0.05450	0.89061	0.13585	0.60558	0.14353	0.40177	0.10718	0.19665	0.05374	0.03289	0.04645	0.00260
0.800	0.200	0.200	0.03371	0.86248	0.11872	0.51529	0.18060	0.28806	0.25012	0.09777	0.32181	0.00482	0.34007	0.00004
0.800	0.200	0.800	0.04737	0.87894	0.14512	0.56223	0.20121	0.33970	0.25796	0.13299	0.32205	0.00970	0.34007	0.00015
0.800	0.200	0.995	0.05209	0.88463	0.15583	0.58132	0.21096	0.36437	0.26266	0.15491	0.32228	0.01553	0.34007	0.00047
0.800	0.800	0.200	0.05034	0.87898	0.17059	0.56346	0.24580	0.34236	0.30829	0.13541	0.34007	0.00983	0.34185	0.00014
0.800	0.800	0.800	0.06498	0.89651	0.20560	0.62199	0.28015	0.41624	0.32795	0.19727	0.34168	0.02292	0.34187	0.00065
0.800	0.800	0.995	0.07004	0.90259	0.22004	0.64618	0.29705	0.45291	0.34073	0.23904	0.34346	0.04107	0.34189	0.00256
0.800	0.995	0.200	0.05610	0.88471	0.19189	0.58327	0.27763	0.36912	0.34581	0.16060	0.36034	0.01670	0.34489	0.00050
0.800	0.995	0.800	0.07108	0.90261	0.23084	0.64697	0.32024	0.45547	0.37701	0.24405	0.36640	0.04409	0.34518	0.00286
0.800	0.995	0.995	0.07627	0.90883	0.24701	0.67347	0.34161	0.49916	0.39849	0.30362	0.37456	0.08963	0.34592	0.01520
0.995	0.200	0.200	0.04031	0.86790	0.15162	0.53187	0.24442	0.30787	0.36922	0.11324	0.56632	0.00765	0.70364	0.00013
0.995	0.200	0.800	0.05425	0.88464	0.18021	0.58133	0.26853	0.36436	0.38007	0.15491	0.56694	0.01553	0.70365	0.00047
0.995	0.200	0.995	0.05907	0.89044	0.19184	0.60148	0.27997	0.39143	0.38660	0.18095	0.56754	0.02497	0.70366	0.00153
0.995	0.800	0.200	0.05733	0.88476	0.20897	0.58401	0.32379	0.37044	0.45688	0.16232	0.62587	0.01739	0.73094	0.00055
0.995	0.800	0.800	0.07228	0.90260	0.24727	0.64618	0.36506	0.45291	0.48613	0.23904	0.63110	0.04107	0.73118	0.00256
0.995	0.800	0.995	0.07745	0.90879	0.26309	0.67194	0.38548	0.49410	0.50536	0.29144	0.63695	0.07428	0.73158	0.01004
0.995	0.995	0.200	0.06322	0.89061	0.23266	0.60558	0.36316	0.40176	0.51602	0.19665	0.70360	0.03289	0.79798	0.00260
0.995	0.995	0.800	0.07853	0.90884	0.27543	0.67347	0.41502	0.49916	0.56464	0.30363	0.72826	0.08963	0.80603	0.01520
0.995	0.995	0.995	0.08383	0.91517	0.29323	0.70179	0.44124	0.54884	0.59880	0.38154	0.76360	0.18923	0.82841	0.08667

Table 1 (*continued*)

Albedo				$\tau_0 = 0.1$		$\tau_0 = 0.5$		$\tau_0 = 1.0$		$\tau_0 = 2.0$		$\tau_0 = 5.0$		$\tau_0 = 10.0$	
W1	W2	W3	W4	REFLEC TIVITY	TRANSMI SSIVITY	REFLEC TIVITY	TRANSMI SSIVITY	REFLEC TIVITY	TRANSMI SSIVITY	REFLEC TIVITY	TRANSMI SSIVITY	REFLEC TIVITY	TRANSMI SSIVITY	REFLEC TIVITY	TRANSMI SSIVITY
0.200	0.200	0.200	0.200	0.01462	0.84684	0.03715	0.47435	0.04393	0.24626	0.04607	0.07274	0.04626	0.00245	0.04626	0.00001
0.200	0.200	0.200	0.800	0.02397	0.85851	0.05145	0.50444	0.05267	0.27658	0.04821	0.09054	0.04628	0.00406	0.04626	0.00003
0.200	0.200	0.200	0.995	0.02716	0.86249	0.05698	0.51610	0.05650	0.28992	0.04936	0.10023	0.04630	0.00554	0.04626	0.00006
0.200	0.200	0.800	0.200	0.02537	0.85840	0.05975	0.50284	0.06292	0.27406	0.05422	0.08816	0.04661	0.00368	0.04627	0.00002
0.200	0.200	0.800	0.800	0.03521	0.87064	0.07727	0.53834	0.07535	0.31337	0.05816	0.11460	0.04668	0.00684	0.04627	0.00007
0.200	0.200	0.800	0.995	0.03857	0.87482	0.08411	0.55224	0.08092	0.33107	0.06038	0.12977	0.04674	0.01011	0.04627	0.00019
0.200	0.200	0.995	0.200	0.02904	0.86235	0.06851	0.51389	0.07128	0.28636	0.05862	0.09662	0.04689	0.00482	0.04627	0.00005
0.200	0.200	0.995	0.800	0.03905	0.87478	0.08738	0.55162	0.08557	0.33003	0.06382	0.12852	0.04704	0.00973	0.04627	0.00018
0.200	0.200	0.995	0.995	0.04248	0.87904	0.09478	0.56644	0.09205	0.34990	0.06683	0.14737	0.04718	0.01538	0.04627	0.00055
0.200	0.800	0.200	0.200	0.02707	0.85839	0.07368	0.50284	0.08658	0.27406	0.07910	0.08816	0.05343	0.00368	0.04668	0.00002
0.200	0.800	0.200	0.800	0.03686	0.87054	0.09047	0.53650	0.09808	0.30988	0.08251	0.11096	0.05348	0.00623	0.04668	0.00006
0.200	0.800	0.200	0.995	0.04021	0.87470	0.09699	0.54959	0.10314	0.32574	0.08434	0.12347	0.05352	0.00858	0.04668	0.00014
0.200	0.800	0.800	0.200	0.03838	0.87052	0.10147	0.53663	0.11382	0.31053	0.09441	0.11161	0.05471	0.00621	0.04669	0.00006
0.200	0.800	0.800	0.800	0.04871	0.88328	0.12238	0.57691	0.13085	0.35849	0.10131	0.14777	0.05492	0.01189	0.04669	0.00020
0.200	0.800	0.800	0.995	0.05224	0.88765	0.13059	0.59274	0.13857	0.38031	0.10526	0.16887	0.05512	0.01785	0.04669	0.00052
0.200	0.800	0.995	0.200	0.04225	0.87467	0.11235	0.54988	0.12610	0.32705	0.10312	0.12520	0.05589	0.00886	0.04671	0.00014
0.200	0.800	0.995	0.800	0.05276	0.88764	0.13502	0.59292	0.14605	0.38113	0.11271	0.17050	0.05643	0.01864	0.04672	0.00056
0.200	0.800	0.995	0.995	0.05636	0.89209	0.14395	0.60991	0.15520	0.40605	0.11841	0.19793	0.05701	0.03036	0.04672	0.00182
0.200	0.995	0.200	0.200	0.03132	0.86235	0.08784	0.51388	0.10533	0.28636	0.09688	0.09662	0.05933	0.00482	0.04719	0.00005
0.200	0.995	0.200	0.800	0.04127	0.87467	0.10565	0.54897	0.11818	0.32470	0.10111	0.12226	0.05942	0.00824	0.04719	0.00013
0.200	0.995	0.200	0.995	0.04467	0.87888	0.11257	0.56263	0.12385	0.34172	0.10340	0.13638	0.05949	0.01141	0.04719	0.00028
0.200	0.995	0.800	0.200	0.04284	0.87467	0.11781	0.54985	0.13675	0.32705	0.11726	0.12521	0.06213	0.00886	0.04728	0.00014
0.200	0.995	0.800	0.800	0.05333	0.88761	0.14014	0.59207	0.15613	0.37914	0.12627	0.16733	0.06259	0.01723	0.04728	0.00048
0.200	0.995	0.800	0.995	0.05692	0.89205	0.14891	0.60870	0.16495	0.40295	0.13147	0.19216	0.06301	0.02615	0.04729	0.00125
0.200	0.995	0.995	0.200	0.04678	0.87889	0.12959	0.56401	0.15107	0.34568	0.12921	0.14229	0.06504	0.01354	0.04744	0.00042
0.200	0.995	0.995	0.800	0.05747	0.89205	0.15385	0.60923	0.17395	0.40482	0.14211	0.19622	0.06638	0.02941	0.04747	0.00170
0.200	0.995	0.995	0.995	0.06112	0.89656	0.16343	0.62712	0.18451	0.43222	0.14988	0.22936	0.06788	0.04925	0.04754	0.00586
0.800	0.200	0.200	0.200	0.02919	0.85849	0.10202	0.50445	0.15623	0.27659	0.22199	0.09054	0.30461	0.00406	0.33591	0.00003
0.800	0.200	0.200	0.800	0.03895	0.87058	0.11866	0.53750	0.16765	0.31167	0.22544	0.11321	0.30467	0.00679	0.33591	0.00008
0.800	0.200	0.200	0.995	0.04229	0.87471	0.12511	0.55033	0.17266	0.32715	0.22729	0.12557	0.30471	0.00929	0.33591	0.00017
0.800	0.200	0.800	0.200	0.04046	0.87053	0.12879	0.53650	0.18177	0.30988	0.23575	0.11096	0.30569	0.00623	0.33593	0.00006
0.800	0.200	0.800	0.800	0.05074	0.88322	0.14932	0.57573	0.19822	0.35583	0.24224	0.14508	0.30589	0.01167	0.33593	0.00020
0.800	0.200	0.800	0.995	0.05426	0.88757	0.15736	0.59112	0.20563	0.37660	0.24589	0.16472	0.30607	0.01731	0.33593	0.00052
0.800	0.200	0.995	0.200	0.04431	0.87465	0.13921	0.54897	0.19308	0.32470	0.24323	0.12226	0.30658	0.00824	0.33594	0.00013
0.800	0.200	0.995	0.800	0.05478	0.88755	0.16138	0.59076	0.21210	0.37598	0.25187	0.16370	0.30701	0.01677	0.33594	0.00048
0.800	0.200	0.995	0.995	0.05836	0.89197	0.17009	0.60722	0.22076	0.39942	0.25690	0.18829	0.30745	0.02659	0.33595	0.00152
0.800	0.800	0.200	0.200	0.04230	0.87061	0.14723	0.53835	0.21843	0.31337	0.28698	0.11460	0.33591	0.00684	0.34170	0.00007
0.800	0.800	0.200	0.800	0.05254	0.88322	0.16707	0.57574	0.23404	0.35583	0.29298	0.14508	0.33609	0.01167	0.34170	0.00020
0.800	0.800	0.200	0.995	0.05604	0.88753	0.17479	0.59031	0.24094	0.37470	0.29622	0.16185	0.33623	0.01613	0.34170	0.00045

Table 1 (*continued*)

Albedo				$\tau_0 = 0.1$		$\tau_0 = 0.5$		$\tau_0 = 1.0$		$\tau_0 = 2.0$		$\tau_0 = 5.0$		$\tau_0 = 10.0$	
0.800	0.800	0.800	0.200	0.05417	0.88326	0.18068	0.57691	0.25658	0.35850	0.31537	0.14777	0.34087	0.01189	0.34186	0.00020
0.800	0.800	0.800	0.800	0.06498	0.89651	0.20560	0.62199	0.28015	0.41624	0.32795	0.19727	0.34168	0.02292	0.34187	0.00065
0.800	0.800	0.800	0.995	0.06867	0.90105	0.21541	0.63977	0.29089	0.44266	0.33519	0.22636	0.34243	0.03454	0.34187	0.00170
0.800	0.800	0.995	0.200	0.05824	0.88760	0.19384	0.59209	0.27395	0.37914	0.33181	0.16732	0.34554	0.01723	0.34210	0.00048
0.800	0.800	0.995	0.800	0.06924	0.90107	0.22095	0.64042	0.30180	0.44472	0.34961	0.23020	0.34765	0.03658	0.34213	0.00187
0.800	0.800	0.995	0.995	0.07300	0.90569	0.23166	0.65956	0.31465	0.47513	0.36029	0.26865	0.34990	0.05990	0.34221	0.00610
0.800	0.995	0.200	0.200	0.04678	0.87477	0.16492	0.55162	0.24644	0.33003	0.32384	0.12852	0.36477	0.00973	0.35006	0.00018
0.800	0.995	0.200	0.800	0.05719	0.88755	0.18609	0.59078	0.26417	0.37597	0.33166	0.16369	0.36514	0.01677	0.35006	0.00048
0.800	0.995	0.200	0.995	0.06075	0.89193	0.19435	0.60605	0.27204	0.39646	0.33589	0.18315	0.36544	0.02329	0.35006	0.00108
0.800	0.995	0.800	0.200	0.05887	0.88763	0.20122	0.59290	0.29124	0.38113	0.36358	0.17050	0.37761	0.01864	0.35139	0.00056
0.800	0.995	0.800	0.800	0.06986	0.90107	0.22801	0.64041	0.31859	0.44471	0.38099	0.23020	0.37969	0.03658	0.35143	0.00187
0.800	0.995	0.800	0.995	0.07362	0.90568	0.23857	0.65918	0.33112	0.47397	0.39115	0.26573	0.38165	0.05580	0.35148	0.00492
0.800	0.995	0.995	0.200	0.06301	0.89204	0.21557	0.60923	0.31191	0.40482	0.38749	0.19622	0.39153	0.02941	0.35403	0.00170
0.800	0.995	0.995	0.800	0.07420	0.90572	0.24478	0.66028	0.34453	0.47762	0.41305	0.27417	0.39806	0.06479	0.35454	0.00694
0.800	0.995	0.995	0.995	0.07803	0.91041	0.25635	0.68055	0.35970	0.51162	0.42867	0.32278	0.40549	0.10996	0.35563	0.02423
0.995	0.200	0.200	0.200	0.03417	0.86249	0.12709	0.51611	0.20521	0.28992	0.31464	0.10023	0.50331	0.00554	0.65052	0.00006
0.995	0.200	0.200	0.800	0.04408	0.87472	0.14469	0.55033	0.21793	0.32714	0.31895	0.12557	0.50342	0.00929	0.65052	0.00017
0.995	0.200	0.200	0.995	0.04746	0.87890	0.15151	0.56362	0.22351	0.34358	0.32125	0.13940	0.50350	0.01275	0.65052	0.00037
0.995	0.200	0.800	0.200	0.04562	0.87470	0.15557	0.54959	0.23395	0.32573	0.33211	0.12347	0.50543	0.00858	0.65058	0.00014
0.995	0.200	0.800	0.800	0.05606	0.88755	0.17733	0.59031	0.25237	0.37469	0.34026	0.16185	0.50581	0.01613	0.65058	0.00045
0.995	0.200	0.800	0.995	0.05963	0.89195	0.18586	0.60630	0.26067	0.39686	0.34486	0.18398	0.50616	0.02396	0.65058	0.00116
0.995	0.200	0.995	0.200	0.04953	0.87888	0.16666	0.56264	0.24671	0.34172	0.34163	0.13638	0.50718	0.01141	0.65065	0.00028
0.995	0.200	0.995	0.800	0.06016	0.89194	0.19020	0.60605	0.26806	0.39646	0.35254	0.18315	0.50802	0.02329	0.65066	0.00108
0.995	0.200	0.995	0.995	0.06380	0.89641	0.19945	0.62317	0.27779	0.42153	0.35890	0.21098	0.50888	0.03699	0.65069	0.00340
0.995	0.800	0.200	0.200	0.04751	0.87481	0.17594	0.55224	0.27733	0.33108	0.40239	0.12977	0.57585	0.01011	0.69027	0.00019
0.995	0.800	0.200	0.800	0.05791	0.88757	0.19705	0.59113	0.29501	0.37660	0.41022	0.16472	0.57624	0.01731	0.69028	0.00052
0.995	0.800	0.200	0.995	0.06146	0.89194	0.20527	0.60629	0.30283	0.39687	0.41445	0.18399	0.57655	0.02396	0.69028	0.00116
0.995	0.800	0.800	0.200	0.05958	0.88764	0.21175	0.59274	0.32102	0.38032	0.44021	0.16887	0.58726	0.01785	0.69138	0.00052
0.995	0.800	0.800	0.800	0.07055	0.90106	0.23835	0.63977	0.34794	0.44266	0.45691	0.22636	0.58909	0.03454	0.69141	0.00170
0.995	0.800	0.800	0.995	0.07431	0.90566	0.24883	0.65833	0.36023	0.47126	0.46656	0.26028	0.59080	0.05216	0.69146	0.00445
0.995	0.800	0.995	0.200	0.06371	0.89204	0.22587	0.60872	0.34102	0.40296	0.46232	0.19215	0.59814	0.02615	0.69300	0.00125
0.995	0.800	0.995	0.800	0.07489	0.90569	0.25483	0.65919	0.37292	0.47397	0.48619	0.26573	0.60305	0.05580	0.69326	0.00492
0.995	0.800	0.995	0.995	0.07871	0.91037	0.26629	0.67920	0.38770	0.50702	0.50059	0.31099	0.60833	0.09176	0.69377	0.01603
0.995	0.995	0.200	0.200	0.05208	0.87904	0.19513	0.56644	0.31015	0.34990	0.45366	0.14737	0.65051	0.01538	0.76518	0.00055
0.995	0.995	0.200	0.800	0.06264	0.89198	0.21770	0.60724	0.33038	0.39941	0.46411	0.18829	0.65146	0.02659	0.76520	0.00152
0.995	0.995	0.200	0.995	0.06626	0.89641	0.22651	0.62317	0.33937	0.42153	0.46979	0.21098	0.65223	0.03699	0.76522	0.00340
0.995	0.995	0.800	0.200	0.06437	0.89209	0.23409	0.60990	0.36188	0.40605	0.50808	0.19794	0.68454	0.03036	0.77899	0.00182
0.995	0.995	0.800	0.800	0.07553	0.90570	0.26274	0.65956	0.39339	0.47513	0.53196	0.26866	0.69013	0.05990	0.77939	0.00610
0.995	0.995	0.800	0.995	0.07934	0.91037	0.27406	0.67920	0.40786	0.50701	0.54596	0.31099	0.69543	0.09176	0.77997	0.01603
0.995	0.995	0.995	0.200	0.06858	0.89656	0.24951	0.62712	0.38588	0.43222	0.54131	0.22936	0.72317	0.04925	0.80874	0.00586
0.995	0.995	0.995	0.800	0.07994	0.91041	0.28081	0.68055	0.42363	0.51162	0.57686	0.32278	0.74184	0.10996	0.81482	0.02423
0.995	0.995	0.995	0.995	0.08383	0.91517	0.29323	0.70179	0.44124	0.54884	0.59880	0.38154	0.76360	0.18923	0.82841	0.08667

REFERENCES

1. M. N. Özişik, *Radiative Transfer*. Wiley, New York (1973).
2. K. M. Case and P. F. Zweifel, *Linear Transport Theory*. Addison-Wesley, Reading, Mass. (1967).
3. C. E. Siewert, *Astrophys. Space Sci.* **58**, 131 (1978).
4. C. E. Siewert and P. Benoist, *Nucl. Sci. Engng* **69**, 156 (1979).
5. P. Grandjean and C. E. Siewert, *Nucl. Sci. Engng* **69**, 161 (1979).
6. C. E. Siewert, J. R. Maiorino and M. N. Özişik, *J. Quantve Spectros. & Radiat. Transf.* **23**, 565 (1980).
7. S. M. Shouman and M. N. Özişik, *J. Quantve Spectros. & Radiat. Transf.* **26**, 1 (1981).
8. M. N. Özişik and S. M. Shouman, *J. Quantve Spectros. & Radiat. Transf.* **24**, 441 (1980).

EFFETS D'UNE VARIATION ECHELON D'ALBEDO SUR LA REFLECTIVITE ET LA TRANSMITTIVITE D'UNE PLAQUE ISOTROPIQUEMENT DIFFUSANTE

Résumé—La réflectivité et la transmittivité d'une plaque à faces parallèles, diffusant isotropiquement, avec une variation échelon d'albedo dans le milieu, sont déterminées pour un rayonnement à incidence isotrope. On présente des résultats exacts sous forme de table pour un large domaine de variation de l'albedo.

EINFLÜSSE EINER SCHRITTWEISEN VERÄNDERUNG DER ALBEDOSTRAHLUNG AUF DIE REFLEXION UND DIE TRANSMISSION EINER ISOTROP STREUENDEN PLATTE

Zusammenfassung—Die Reflexion und die Transmission einer isotrop streuenden plan-parallelen Platte bei schrittweiser Veränderung einer einfach streuenden Albedostrahlung innerhalb des Mediums werden für eine isotrop einfallende Strahlung bestimmt. Es werden genaue Ergebnisse in tabellarischer Form angegeben, die einen breiten Bereich der Albedostrahlung abdecken.

ВЛИЯНИЕ СТУПЕНЧАТОГО ИЗМЕНЕНИЯ АЛЬБЕДО НА ОТРАЖАТЕЛЬНУЮ И ПРОПУСКАТЕЛЬНУЮ СПОСОБНОСТИ ИЗОТРОПНО РАССЕИВАЮЩЕЙ ПЛИТЫ

Аннотация—Для изотропно падающего излучения определены отражательная и пропускательная способности изотропно рассеивающей плоско-параллельной плиты при ступенчатом изменении однократного альбедо рассеяния в среде. Затабулированы точные результаты для широкого диапазона альбедо.